

Suggestion for a proof of Philipp Emanuel Weidmann's conjecture concerning A027983 as stated in The Sequencer OEIS survey

$$a_n = A027960_{n,n+1} + A027960_{n,n+2} + \cdots + A027960_{2n} \text{ (see also the name of A027983)}$$

$A027960_{n,k}$ conveniently will be abbreviated by $T_{n,k}$.

$$\text{Conjecture: } a_n = a_{n-1} + a_{n-2} + 2^n$$

$$a_3 = T_{3,4} + T_{3,5} + T_{3,6} = 8 + 5 + 1 = 14$$

$$a_2 = T_{2,3} + T_{2,4} = 4 + 1 = 5$$

$$a_1 = T_{1,2} = 1$$

$$a_2 + a_1 + 2^3 = 5 + 1 + 8 = 14 = a_3 \quad \text{For } n=3 \text{ the conjecture is true.}$$

$$\text{To show: } a_{n+1} = a_n + a_{n-1} + 2^{n+1} \text{ if } a_n = a_{n-1} + a_{n-2} + 2^n$$

Proof:

1. $a_n = a_{n-1} + a_{n-2} + 2^n$ (initial step)
2. $a_n = 2a_{n-1} + T_{n-1,n-1}$ (because of definition of A027960)
3. $T_{n,n}$ are the Lucas Numbers (see crossrefs of A027960)
4. $T_{n,n} = T_{n-1,n-1} + T_{n-2,n-2}$ (from 3, see name of the Lucas Numbers, A000032)
5. $a_{n+1} = 2a_n + T_{n,n}$ (from 2)
6. $a_{n+1} = 2(a_{n-1} + a_{n-2} + 2^n) + T_{n-1,n-1} + T_{n-2,n-2}$ (from 1, 4 and 5)
7. $a_{n+1} = 2a_{n-1} + T_{n-1,n-1} + 2a_{n-2} + T_{n-2,n-2} + 2^{n+1}$ (from 6)
8. $a_{n+1} = a_n + a_{n-1} + 2^{n+1}$ (from 2)

For $n > 2$ the conjecture seems to be proved.